STATISTICAL ANALYSIS IN EXPERIMENTAL PARTICLE PHYSICS

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EXERCISE 1

#1: BINOMIAL DISTRIBUTION FROM PRINCIPLE

- Let's generate a binomial distribution based on the "principle" of binomial itself, e.g. accept event with a fixed probability $p=10\%$ for a given $N=100$ trials:

```
{
    TRandom3 rnd;

    int count = 0;
    for(int i=0;i<100;i++)
        if (rnd.Uniform()<0.1) count++;

    cout << count << endl;
}
```

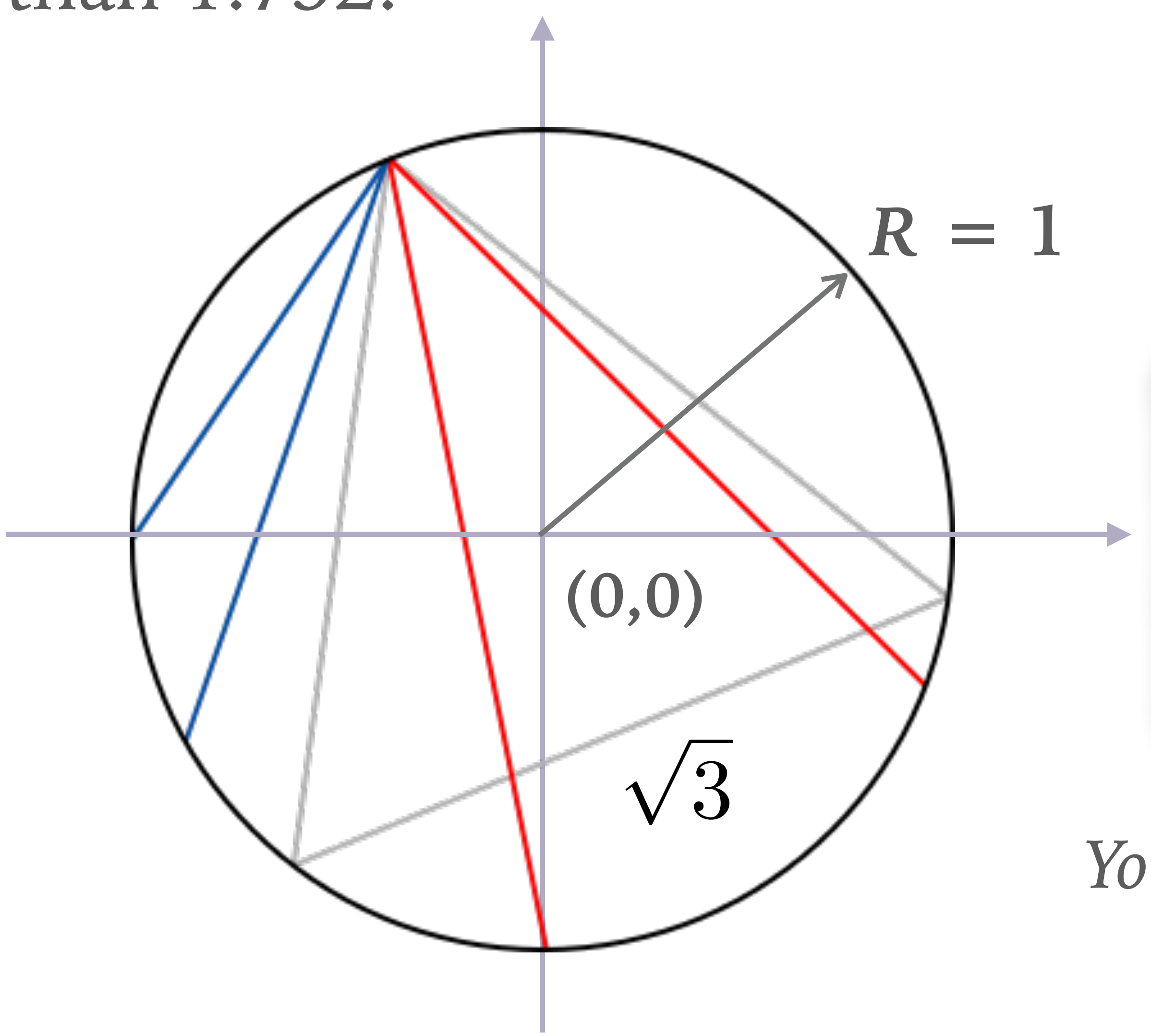
- The code above will give one single "count" which follows the binomial distribution. Please repeat the generation for 10,000 times and prove the variance follows $Np(1-p) = 9$.

#2: SUM OF TWO BINOMIAL VARIABLE

- Based on the previous exercise, generate two random binomial variables:
  - $N = 100, p = 10\%$
  - $N' = 200, p = 25\%$
- Verify the variance of the sum of the two random binomial variables is smaller than an averaged binomial distribution:
  - $N = 300, p = 20\%$

#3: BERTRAND'S PARADOX

- Consider a circle, centered at (0,0) with radius 1. Randomly choose two points on the circumference of the circle and calculate the distance between them. Draw the distribution for the distance of the chords, and verify 1/3 of the chord is longer than 1.732.

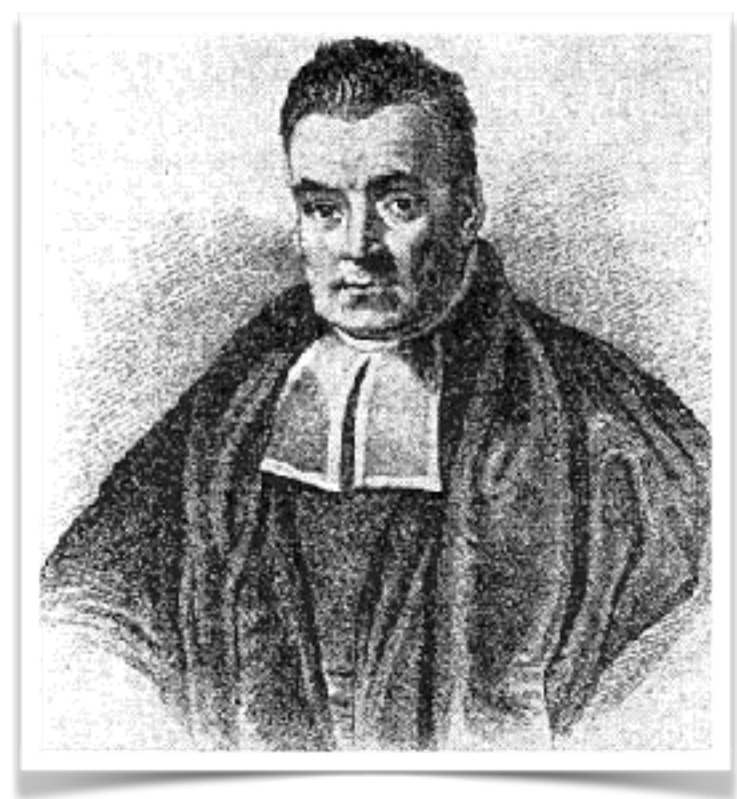


What if the you choose random two points within the circle and produce a chord? What would be the fraction in this case?

You can try the other two proposed methods.

#4: FREQUENTIST VS BAYESIAN

- Find **10 different probabilities in daily life** and see if they can be defined either by **frequentist** or by **Bayesian**.
- Just like the example given in the main lecture, the “raining probability” can be only defined by Bayesian.



Thomas Bayes

#5: CONDITIONAL PROBABILITY

- Consider someone rolls **two fair dice**, and we must predict the outcome (the sum of the two upward faces), which should be within **[2, 12]**.
- Let $\mathbf{D_1}$ and $\mathbf{D_2}$ are the value rolled on die 1 and die 2, respectively.
- Suppose event A is $\mathbf{D_1+D_2 > 8}$, and event B is $\mathbf{D_1 = D_2}$, show this example fulfill the condition below:

$$P(A \text{ and } B) = P(A|B)P(B) = P(B|A)P(A)$$

#6: BAYERS THEOREM FOR DISCRETE EVENTS

- The entire output of a factory is produced on three machines. The three machines account for different amounts of the factory output, namely **25%**, **35%**, and **40%**.
- The fraction of defective items produced is this: for the first machine, **4%**; for the second machine, **2%**; for the third machine, **1%**.
- If an item is chosen at random from the total output and is found to be defective, what is the probability that it was produced by the third machine?

#7: PRACTICE WITH PDF

- Consider a physics process which produce a probability distribution proportional to $1+t^2+\exp(-t)$, while t is in the range of $[0,1]$.
- Derive the **probability density function** for this physics process and produce a plot of it.
- Derive the **Cumulative distribution** for this PDF and plot it.

#8: BAYERS THEOREM FOR CONTINUOUS VARIABLE

- Consider a particle experiment which can measure the mass of an unknown new particle. Suppose the true mass of the particle is denoted by M, which is known to be within the range of 1-3 GeV with uniform probability density.
- The detector resolution has been estimated to be 0.1 GeV, and can be perfectly modeled by a Gaussian distribution.
- Suppose the measured mass is 1.5 GeV, plot **the posterior probability density distribution for a given true mass M, ie. P(M|1.5 GeV)**.
- If the prior density is not uniform but also a Gaussian of mean 2 GeV and width 0.5 GeV (e.g. constrained by other studies), what is the posterior density distribution in this case?